A well-balanced non-hydrostatic pressure model for steady and unsteady shallow flows

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Abstract A depth-averaged system for free surface flow based in non-hydrostatic pressure (NHP) assumptions is presented and analyzed for the numerical simulation of open channel flows when simulating both steady and unsteady. The NHP model is solved by means of a one-dimensional explicit-implicit finite volume method. A well-balanced scheme must be designed for these models to solve some numerical problems. Quasi-analytical and experimental results validate the NHP model.

Keywords: non-hydrostatic pressure, finite-volume, dispersive, steady, well-balanced

Equations and numerical scheme

The 1D depth-averaged non-hydrostatic system of equations can be written (Bristeau *et al*. (2015)):

SWE

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = 0$$

$$\frac{\partial (hu)}{\partial t} + \frac{\partial}{\partial x}\left(hu^2 + \frac{1}{2}gh^2\right) + gh\frac{\partial z_b}{\partial x} = \underbrace{-\frac{1}{2}\left(h\frac{\partial p_{nh}}{\partial x} + p_{nh}\frac{\partial (h+2z_b)}{\partial x}\right)}_{\text{Non-hydrostatic terms}}$$

NHP

$$\frac{\partial w}{\partial t} + \frac{\partial (wu)}{\partial x} = \frac{p_{nh}}{h}$$

CONTINUITY EQ.

$$h\frac{\partial (hu)}{\partial x} + 2hw + (hu)\frac{\partial}{\partial x}(h+2z_b) = 0$$

The system is solved in two steps by means of the Fractional step procedure (Stelling and Zijlema (2003))

$h^n, (hu)^n, w^n, p_{nh}^n$

↓

Solve SWE

↓

$h^*, (hu)^*, w^*$

↓

Force continuity eq. with p_{nh} as unknown

↓

p_{nh}^{n+1}

↓

Update variables with p_{nh}

↓

$h^{n+1}, (hu)^{n+1}, w^{n+1}$

The hydrostatic part of the system is solved my means of a first order FV numerical scheme (Murillo and Garcia-Navarro (2010)), obtaining h*, (hu)* and w*, assuming $p_{nh} = 0$

The implicit Pressure Correction Method provides the new p_{nh} distribution.

Finally, this non-hydrostatic distribution is used to update the final values of the new state, h^{n+1} and $(hu)^{n+1}$.

Results

Unsteady wave propagation

The same model reproduces properly experimental waves measured on an open channel Beji and Battjes (1993)

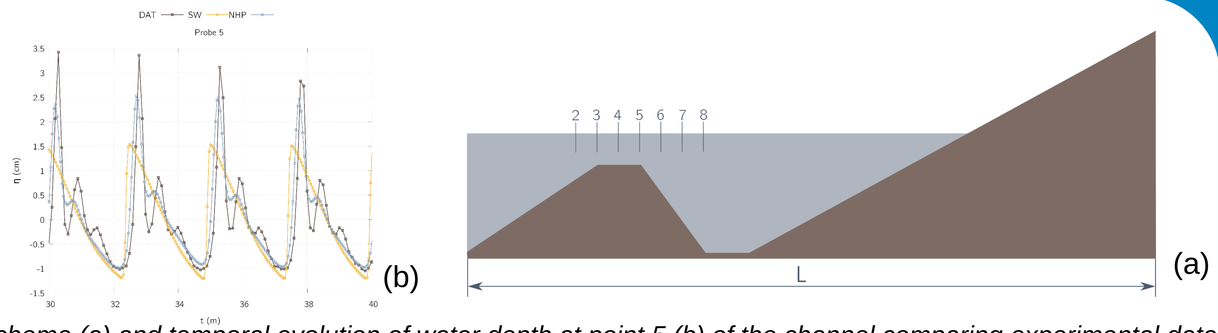

Fig. 1: Experimental layout scheme (a) and temporal evolution of water depth at point 5 (b) of the channel comparing experimental data (brown), simulation results with a SW model (orange) and with the NHP model (blue).

Steady quasi-analytical solution

A quasi-analytical solution of the system can be found in Bristeau *et al.* (2015). The fractional step procedure presents numerical problems of spatial discharge conservation:

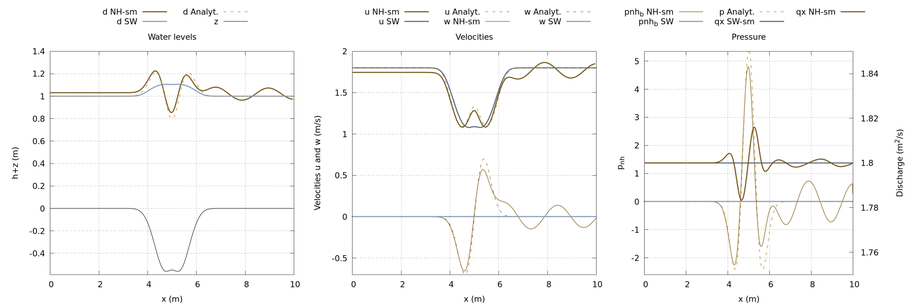

Fig. 2: Spatial distribution of flow variables in a steady state over irregular topography, z.

Conclusions

The application of the fractional step procedure to solve a NHP model is very effective and robust, and presents accurate results when dealing with unsteady wave propagation, as can be seen in Fig. 1. However, the same method applied to a steady case shows the necessity of an equilibrium analysis of the scheme, since the numerical fluxes after the corrector step get unbalanced and the steady discharge is not able to remain spatially constant.

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