Optimization of transport velocity for ALE-SPH scheme

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Abstract Smoothed Particle Hydrodynamic (SPH) is a Lagrangian meshless numerical method suited to simulate complex flows with fragmented interfaces and/or moving objects. In the present work, a novel formulation has been developed in the framework of Arbitrarily Lagrangian-Eulerian SPH schemes where the transport velocity, $\boldsymbol{v}_0$ can be defined differently to best fit the problem. A new formulation for $\boldsymbol{v}_0$ is proposed and compared with literature references using the inviscid Taylor Green vortex test case. The new algorithm preserves good quality in particle distribution while keeping the computational effort reasonable and it can be applied to a wider range of fluid simulations.

Keywords: ALE-SPH: Arbitrary Lagrangian-Eulerian Smoothed Particle Hydrodynamics, PST: Particle Shifting Technique.

Methods

In ALE-SPH (Vila, 1999) scheme the position of generic particle i^{th} is updated in time using the transport velocity $\boldsymbol{v}_{0i}$ and, in a quasi-Lagrangian method (Oger et al, 2016), $\boldsymbol{v}_0$ is defined as the fluid velocity, $\boldsymbol{v}$, with a shifting correction term $\delta\boldsymbol{v}$

$$\frac{\partial \boldsymbol{x}_i}{\partial t} = \boldsymbol{v}_{0i} = \boldsymbol{v}_i - \delta\boldsymbol{v}_i \qquad [1]$$

The correction taken as reference (Neuhauser, 2014), is:

$$\delta\boldsymbol{v}_i = \sum_{j\in D_i} \omega_j \left[\frac{p_{ref}}{\rho_0} + \frac{c_0}{2}\left(\boldsymbol{v}_{0i} - \boldsymbol{v}_{0j}\right)\cdot \boldsymbol{n}_{ij}\right]\nabla W_{ij} \qquad [2]$$

A new formulation for $\delta\boldsymbol{v}$ introduces a fictitious pressure field based on the particle volume variation, $\frac{\omega_0}{\omega_i}$.

$$\delta\boldsymbol{v}_i = \sum_{j\in D_i} \omega_j \left[\frac{1}{\rho_0}\frac{P_i^F + P_j^F}{2} + \frac{c_0}{2}\left(\boldsymbol{v}_{0i} - \boldsymbol{v}_{0j}\right)\cdot \boldsymbol{n}_{ij}\right]\nabla W_{ij} \qquad [3]$$

with

$$P^F = \frac{\rho_0 c_0^2}{\gamma} + \frac{\rho_0 c_0^2}{\gamma}\left[\left(\frac{\omega_0}{\omega}\right)^{\gamma} - 1\right] \qquad [4]$$

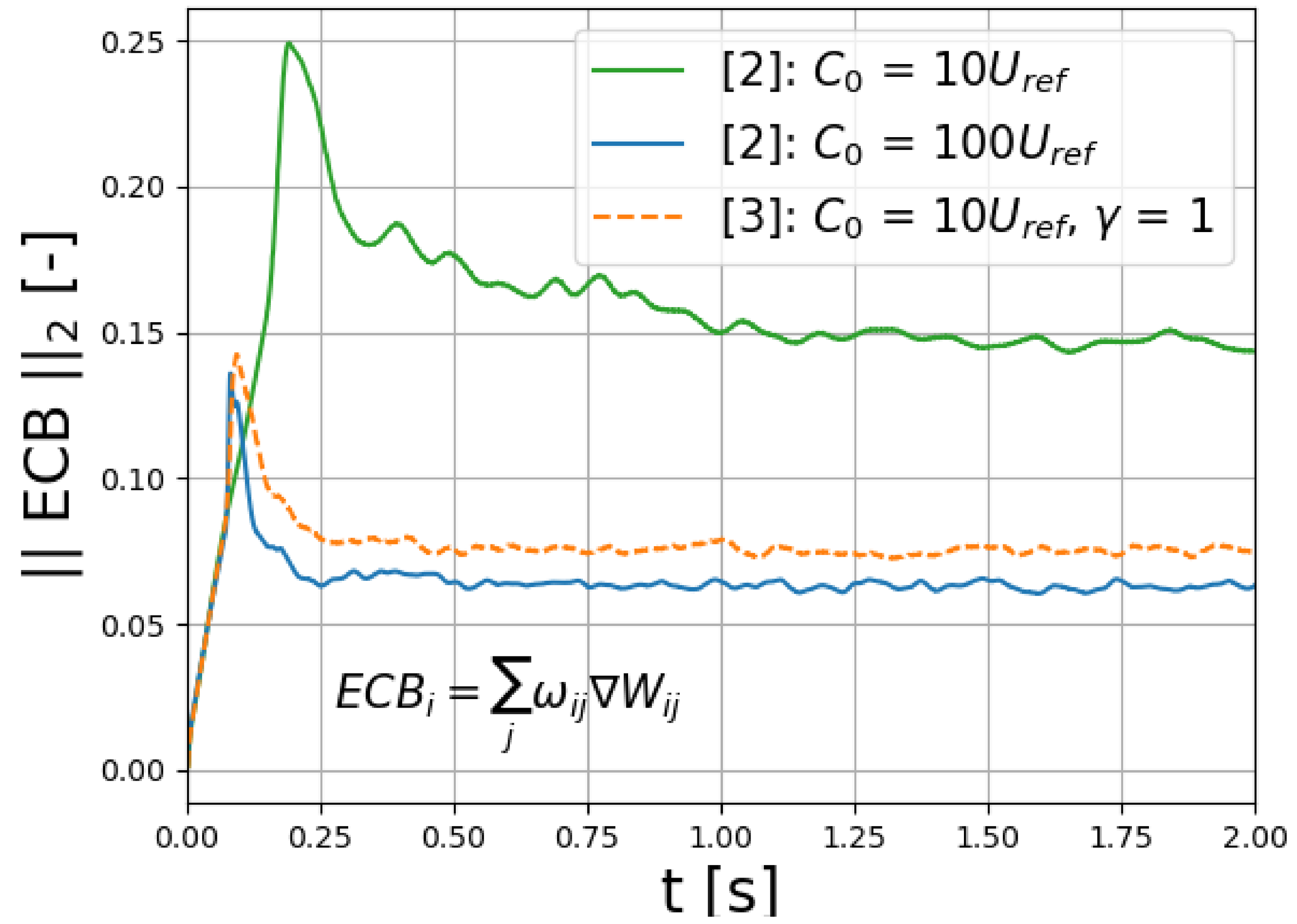

Figure 1: Error Closed Box (ECB) L_2 Norm

Results

The inviscid Taylor Green Vortex (TGV) test case has been simulated to evaluate the capability of [3] to maintain quality of particle distribution. This property can be assessed briefly using the Error Closed Box (ECB) L_2 norm (i.e. measurement for concentration gradient) as illustrated in Figure 1. The ECB has zero value for particles arranged in a cartesian grid and it increases for particles distributed not homogeneously.

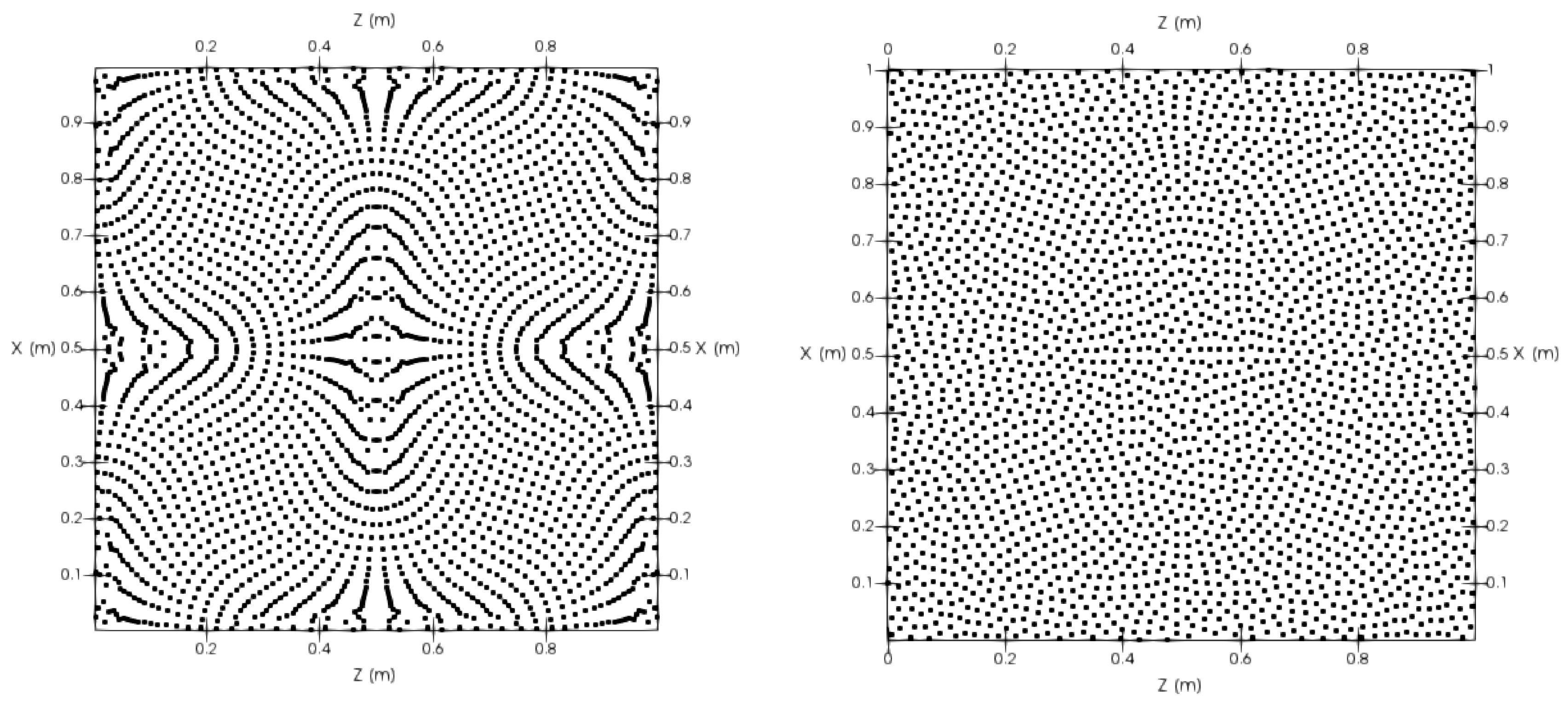

Figure 2: TGV simulation, $U_{ref} = 1\frac{m}{s}$. Particle position at $t = 0.2s$. (left) [2] $c_0 = 10U_{ref}$, (right) [3] $c_0 = 10U_{ref}, \gamma = 1$

The simulation has been conducted for a total physical time of $2s$. The total computational time for three different cases is reported in Table 1.

Table 1: Computational time.

Case	Computational time[s]
[2]: $c_0 = 10U_{ref}$	29.90
[2]: $c_0 = 100U_{ref}$	258.59
[3]: $c_0 = 10U_{ref}, \gamma = 1$	29.37

Conclusions

Results obtained using a weight-based formulation for $\boldsymbol{v}_0$, show that the proposed method is capable to maintain a homogeneous particle distribution. The shifting correction introduced with [3] has a stronger impact on particle displacement in respect with [2] using the same value of c_0. The particles, following the counter-rotating trajectories in TGV, tend to create hole or cluster in the domain but the new PST applied preserves constant particle concentration that is necessary to guarantee accuracy of the SPH interpolation, without introducing any significant computational overheads. This method can be implemented without any difficulties into different ALE-SPH numerical models. In future works this formulation will be used to simulate fluid flow in Pelton turbine.

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