2. Solution 1D System

a)

$$\frac{\partial^2 h}{\partial x^2} = -\frac{1}{K}(w_1 x + w_0) \qquad h(0) = h_0 \quad h(L) = h_L$$

$$\frac{\partial h}{\partial x} = -\frac{1}{K}(\frac{1}{2}w_1 x^2 + w_0 x) + a_1$$

$$h(x) = -\frac{1}{K}(\frac{1}{6}w_1 x^3 + \frac{1}{2}w_0 x^2) + a_1 x + a_0$$

$$h(0) = h_0 = a_0$$

$$a_0 = h_0$$

$$h(L) = h_L = -\frac{1}{K}(\frac{1}{6}w_1 L^3 + \frac{1}{2}w_0 L^2) + a_1 L + h_0$$

$$a_1 = \frac{1}{L}(h_L - h_0 + \frac{1}{K}(\frac{1}{6}w_1 L^3 + \frac{1}{2}w_0 L^2))$$

b)

$$h(x) = -\frac{1}{K}(\frac{1}{6}w_1 x^3 + \frac{1}{2}w_0 x^2) + a_1 x + a_0$$

$$q(0) = q_0 = -K\frac{\partial h}{\partial x} = (\frac{1}{2}w_1 x^2 + w_0 x) - K\ a_1$$

$$a_1 = -\frac{q_0}{K}$$

$$h(L) = h_L = -\frac{1}{K}(\frac{1}{6}w_1 L^3 + \frac{1}{2}w_0 L^2) - \frac{q_0}{k}L + a_0$$

$$a_0 = h_L + \frac{1}{K}(\frac{1}{6}w_1 L^3 + \frac{1}{2}w_0 L^2) + \frac{q_0}{K}L$$

c)

$$h(x) = -\frac{1}{K}(\frac{1}{6}w_1 x^3 + \frac{1}{2}w_0 x^2) + a_1 x + a_0$$

$$q(0) = q_0 = -K\frac{\partial h}{\partial x} = -K\ a_1$$

$$q(L) = q_l = -K\frac{\partial h}{\partial x} = (\frac{1}{2}w_1 L^2 + w_0 L) - K\ a_1$$

-> no equation for a_0, problem cannot be solved

I)

$$a_0 = 12$$

$$a_1 = \frac{1}{4}(18 - 12) = 1.5$$

$$h(\frac{L}{2}) = 1.5 * \frac{4}{2} + 12 = 15$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -3 & 6 & -3 & 0 & 0 \\ 0 & -3 & 6 & -3 & 0 \\ 0 & 0 & -3 & 6 & -3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \\ 0 \\ 18 \end{bmatrix}$$

II)

$$a_0 = 12$$

$$a_1 = \frac{1}{4}(18 - 12 + \frac{1}{3}0.5 * 2 * 4^2) = \frac{17}{6}$$

$$h(\frac{L}{2}) = -\frac{1}{3}(\frac{1}{2}2 * 2^2) + \frac{17}{6} * 2 + 12 = 16.\overline{3}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -3 & 6 & -3 & 0 & 0 \\ 0 & -3 & 6 & -3 & 0 \\ 0 & 0 & -3 & 6 & -3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{bmatrix} = \begin{bmatrix} 12 \\ 2 \\ 2 \\ 2 \\ 18 \end{bmatrix}$$

III)

$$a_1 = -\frac{6}{3} = -2$$

$$a_0 = 18 + 8 = 26$$

$$h(\frac{L}{2}) = -2 * 2 + 26 = 22$$

$$\begin{bmatrix} 3 & -3 & 0 & 0 & 0 \\ -3 & 6 & -3 & 0 & 0 \\ 0 & -3 & 6 & -3 & 0 \\ 0 & 0 & -3 & 6 & -3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 0 \\ 0 \\ 18 \end{bmatrix}$$

IV)

$$a_1 = -\frac{6}{3} = -2$$

$$a_0 = 18 + \frac{16}{3} + 8 = \frac{94}{3}$$

$$h(\frac{L}{2}) = -\frac{4}{3} - 2 * 2 + \frac{94}{3} = 26$$

$$\begin{bmatrix} 3 & -3 & 0 & 0 & 0 \\ -3 & 6 & -3 & 0 & 0 \\ 0 & -3 & 6 & -3 & 0 \\ 0 & 0 & -3 & 6 & -3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 2 \\ 2 \\ 18 \end{bmatrix}$$